

TRAJECTORY BASED COMPUTATIONAL AEROTHERMODYNAMIC INVESTIGATION TOOL SPARTA FOR PLANETARY ENTRY PROBES

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ABSTRACT

A platform independent GUI based tool SPARTA (*Space probe Aerothermodynamics Reentry Trajectory Analysis*) has been developed to study aerodynamic heating and to do trajectory analysis. The dimensionless heat transfer coefficient Stanton Number is analyzed based on the aerodynamic heating rate.

The Apollo command module and the space shuttle provide examples of successful manned reentry vehicles. The Apollo Command Module reenters the earth's atmosphere at 36,175 ft/s. The Space Shuttle reenters at about 24500 ft/s. A comprehensive database of different ballistic reentry vehicles has been developed. This includes vehicle dimensions, trajectory data for all the Apollo capsules that has been flown and ballistic reentries have been modeled for comparative data analysis.

Empirical models of appropriate planetary atmosphere are modeled as subroutines that are called in the trajectory engine to calculate pressure, density, temperature, Reynolds number and speed of sound as a function of altitude. The vehicular Flight path angle as a function of velocity and altitude to compute the trajectory of the vehicle as it traverses through the atmosphere has been modeled. A fourth order Runge-Kutta integration is employed for trajectory calculations. Transport properties such as coefficient of viscosity, thermal conductivity have been modeled using the Sutherland's law. An empirical correlation for stagnation point heat transfer has been modeled using Fay-Riddell theory.

NOMENCLATURE

h	enthalpy (J/kg)
T	Temperature (K)
P	Pressure (N/m ²)
V	freestream velocity (m/s)
q	heat transfer rate W/m ²
M	Mach number
Re	Reynolds number
St	Stanton number
K_n	Knudsen number
P_r	Prandtl's number
m	entry mass (Kg)
s	base area of the capsule (m ²)
R_b	fore body radius (m)
t	time (sec)
C_p	Specific heat at constant pressure (kJ/kg.K)
C_v	Specific heat at constant volume

GREEK

α	Angle of attack
ρ	Freestream density (kg/m ³)
β	Ballistic Coefficient (kg/m ²)
η	Coefficient of Viscosity
κ	Thermal conductivity (W/m-k)
Υ	Ratio of specific heats.
ε	Emissivity of the material
σ	Stefan-Boltzmann's constant
γ	Flight path angle (°)
δ	Shock Stand-Off distance γ

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1. Introduction

As flight-testing of entry probes is generally prohibitively expensive, mission planners must rely on Computational Fluid Dynamics (CFD) simulations in their design. CFD can be used to simulate entry vehicle aerodynamics and aerothermodynamics for flight regimes of interest. The current trend is to rely more and more on physical and thermo-chemical models of computational simulations. It is desirable when designing a new planetary probe or planning for the use of an existing design to rely on computational predictions of vehicle performance in addition to wind tunnel and flight data. Essential aspects to assure the accuracy and reliability of these solutions are the selection of the grid topology, grid resolution, and grid quality.

A comprehensive database of existing ballistic reentry vehicles has been developed from the Planetary Mission Entry Vehicles manual^[1] and is provided in the SPARTA framework. This includes vehicle dimensions, trajectory data for all the capsules that has been flown and ballistic reentries have been modeled for comparative data analysis. The SPARTA GUI provides capabilities to choose from a list of flight vehicle geometric information and entry trajectories.

2. Atmospheric Profile

SPARTA (*Space probe Aerothermodynamics Reentry Trajectory Analysis*) design environment also links the trajectory code to appropriate planetary empirical models depending on the planetary probe that is chosen. The 1976 US Standard Atmosphere for Earth called GAME – General Atmospheric Model for Earth is modeled as a subroutine that calculates pressure, density, temperature, Reynolds number and speed of sound as a function of altitude. Martian and Venusians atmospheric profiles are also modeled as subroutines that return the pressure, density and temperature profiles.

3. Trajectory Analysis

A fourth order Runge-Kutta integration is employed for trajectory calculations to advance the solution of the differential equations to solve the following set of equations (1-a) to (1-d).

$$\frac{dV}{dh} = \frac{g[\frac{Q}{\beta} - \sin(\gamma)]}{V \sin(\gamma)} \quad (1-a)$$

$$\frac{d\gamma}{dh} = \frac{\cos(\gamma)[-g + \frac{V^2}{R_e + h}]}{V^2 \sin(\gamma)} \quad (1-b)$$

$$\frac{dt}{dh} = \frac{-1}{V \sin(\gamma)} \quad (1-c)$$

$$\frac{dr}{dh} = R_e \frac{d\theta/dt}{dh/dt} = \frac{-R_e \cos(\gamma)}{(R_e + h) \sin(\gamma)} \quad (1-d)$$

Figure 3[a-e] shows the variation of flight velocity, Reynolds number, Stagnation point pressure and Deceleration against Atmospheric Altitude during hypersonic ballistic reentry. Three different flight vehicle trajectories have been analyzed for different values of flight path angle (γ) and Ballistic Coefficient (β). Table 1 and table 2 shows the database of flight vehicles of initial trajectory data and vehicle dimensions data that's used in the GUI and the program SPARTA to do the trajectory calculations. Apollo Capsules AS201, Apollo 4 and Apollo 6 were chosen for trajectory analysis since they have identical vehicle dimensions and the plots. Figure 3 shows Apollo AS201 with an initial flight path angle of -8.58° and Apollo 4 with an initial flight path angle of -6.92° and Apollo 6 with an initial flight path angle of -5.9° . Figures 3[a-e] show different trajectories since their initial flight path angles and ballistic coefficients are different for these flight vehicles. The maximum deceleration appears at about 50,000 ft altitude close to the height of maximum stagnation point heating load of 700 BTU/ft²-sec at 52,000 ft altitude.

3.1. Atmospheric Probe Model

The probe model developed for this trajectory is a point-mass model with two translations and one rotation around a spherical planet. It integrates the equations of motion of a vehicle on a ballistic entry trajectory so that no lift is generated and the body acts only on gravity. Figure 1 shows the various aerodynamic forces acting on the body

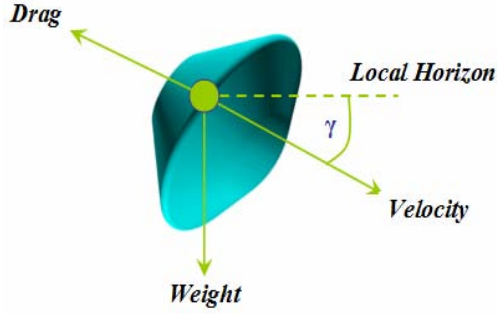
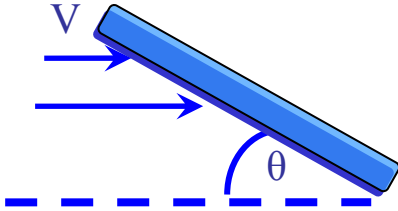


Figure 1: Aerodynamic forces acting on the Ballistic entry vehicle

The vehicle model is build from a number of parameters defining the geometry of the probe including body diameter, cone half-angle, nose and shoulder radius. The aerodynamic properties of the probe are subsequently derived from the geometry of the vehicle. Modified Newton Flow Theory (NFT) is used to evaluate the pressure coefficient C_p around the body and derive the drag coefficient C_d of the configuration. The Modified NFT states that

$$C_p = C_{pmax} \sin^2 \theta \quad (2)$$



Where θ is the angle between the flow and the body surface and C_{pmax} is evaluated as the maximum pressure coefficient found behind a normal shockwave at the stagnation point. The Ballistic Coefficient of the probe is derived from the aerodynamic model as per equation (3)

$$\beta = m/(C_d)(A) \quad (3)$$

where m is the mass and A is the section area of the body. The higher the ballistic coefficient, the higher the heat and deceleration loads. Once the ballistic coefficient is determined, SPARTA provides a number of entry profiles for various velocities (V_e) and entry angles (γ_e)

3.2 Stagnation Point Heating Analysis

An empirical correlation for stagnation point heat transfer rate for Earth has been modeled using Fay-Riddell theory [2] and Sutton-Grave correlation [3] as described in equation (4) which gives the heating rate per unit area as

$$\dot{q} = C_p^N V^M \quad (4)$$

where \dot{q} is the heat-transfer rate into the flight body, per unit area, ρ is the freestream density, and V is the flight velocity. The constants $M=3$ and $N=0.5$ give the heating rate in W/cm^2 if the velocity is given in m/s and the density in kg/m^3 .

$$C = (1.83 \times 10^{-8}) (r_n)^{-1/2} (1-g_w) \quad (5)$$

where r_n is the body nose radius, in meters and g_w is the ratio of wall enthalpy (h_w) to total enthalpy (h_0). From thermodynamics, $h_w = c_{pw} T_w$. The Total enthalpy is $h_0 = h_a + \frac{1}{2} V^2$, where h_a is the local enthalpy of the atmosphere. However, for reentry, h_a is usually smaller than $\frac{1}{2} V^2$, and can be neglected [3]. Thus,

$$g_w = \frac{C_{pw} T_w}{\frac{1}{2} V^2} \quad (6)$$

Therefore, from equations (4), (5) and (6), we have,

$$\dot{q} = (1.83 \times 10^{-8}) \left[\frac{\rho}{r_n} \right] V^3 \left(1 - \frac{C_{pw} T_w}{\frac{1}{2} V^2} \right) \quad (7)$$

The maximum heating rate for an entry profile is evaluated at the stagnation point from the Sutton-Grave correlation as in equation (8).

$$\dot{q}_s = k \sqrt{\frac{\rho}{r_n}} \left(\frac{V_\infty}{1000} \right)^3 \quad (8)$$

Transport properties such as coefficient of viscosity, thermal conductivity have been modeled using the Sutherland's law.

4. SPARTA Database

SPARTA database framework has Trajectory, TPS and Geometry data stored for each probe in the database. In addition to the capsule shapes, base areas, nose radii, total and payload masses and the ballistic coefficients of the probes are stored in the database.

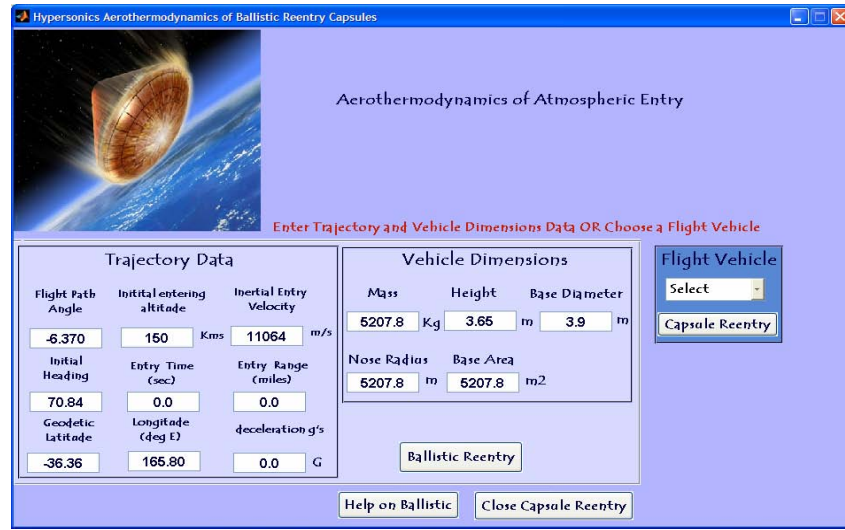


Figure 2: SPARTA Graphical User Interface

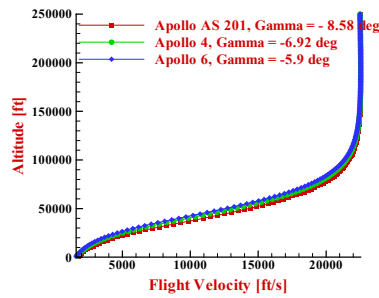


Fig 3(a) Flight Velocity with Altitude

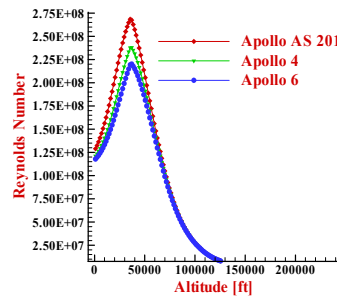


Fig 3(b) Reynolds number Vs Altitude

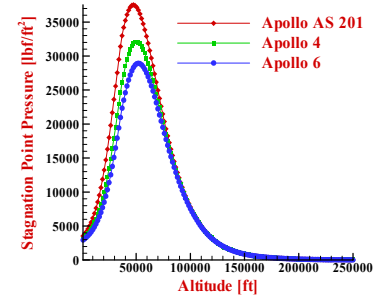


Fig3(c) Stagnation Point Pressure Vs Altitude

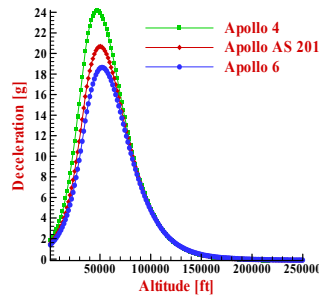


Fig 3(c) Deceleration Vs Altitude

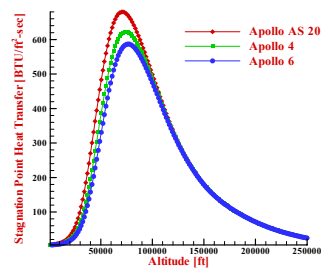


Fig 3(d) Stagnation Point Heat Vs Altitude

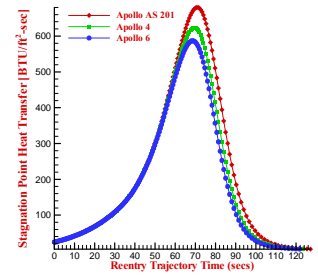


Fig 3(e) Stagnation Point Heat Vs Time

Figure 3[a-e] shows the variation of flight velocity, Reynolds number, Stagnation point pressure and Deceleration against Atmospheric Altitude during reentry. Three different flight vehicle trajectories have been analyzed for different values of flight path angle (γ) and Ballistic Coefficient (β). Apollo Capsules were chosen for trajectory analysis since they have

identical vehicle dimensions and the plots 3[a-e] show different trajectories since their initial flight path angles and ballistic coefficients are different for these flight vehicles. A sample trajectory data is shown in Table 1. SPARTA database has flight vehicles of initial trajectory data and vehicle dimensions data that's used in the GUI for trajectory calculations.

Table 1: Sample Output of Trajectory Calculated by SPARTA

Trajectory Analysis for Flight Vehicle: Apollo 4						
Altitude (ft)	Velocity (ft/sec)	Flight Path Angle γ (deg)	Mach Number	Reynolds Number	Stag Point Pressure (lbf/ft ²)	Stag Point Heat Transfer rate • Q (Btu/ft ² -sec)
250000.00	22500.00	12.00	23.83	25524.59	30.70	168.44
249500.00	22500.69	12.00	23.82	26092.81	31.42	170.43
249000.00	22501.37	12.00	23.80	26672.87	32.16	172.43
248500.00	22502.06	12.01	23.78	27265.00	32.92	174.46
248000.00	22502.75	12.01	23.77	27869.44	33.69	176.50
247500.00	22503.43	12.01	23.75	28486.43	34.48	178.57
247000.00	22504.12	12.01	23.73	29116.20	35.28	180.66
246500.00	22504.80	12.01	23.72	29759.02	36.11	182.77
246000.00	22505.49	12.02	23.70	30415.12	36.95	184.90
245500.00	22506.17	12.02	23.69	31084.77	37.81	187.06
245000.00	22506.86	12.02	23.67	31768.23	38.69	189.23
244500.00	22507.54	12.02	23.65	32465.75	39.59	191.43
244000.00	22508.23	12.02	23.64	33177.62	40.51	193.65
243500.00	22508.91	12.03	23.62	33904.10	41.44	195.90
243000.00	22509.59	12.03	23.60	34645.48	42.40	198.16
242500.00	22510.28	12.03	23.59	35402.04	43.38	200.45
242000.00	22510.96	12.03	23.57	36174.07	44.38	202.76
241500.00	22511.64	12.03	23.56	36961.87	45.40	205.10
241000.00	22512.32	12.04	23.54	37765.73	46.45	207.46
240500.00	22513.01	12.04	23.53	38585.96	47.51	209.85
240000.00	22513.69	12.04	23.51	39422.87	48.60	212.25

5. Planetary vehicle web application architecture

A Relational database management system SPARTA Probe DB has been developed and integrated with .NET Framework for web accessibility as shown in figure (4). RDBMS

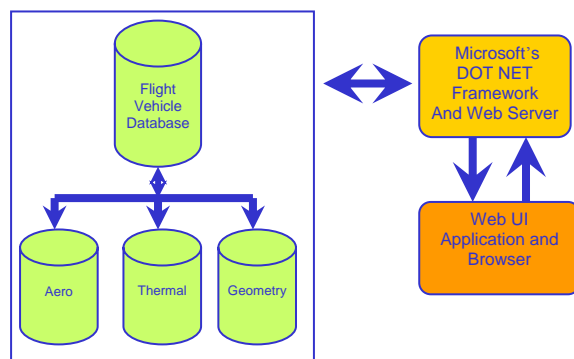


Fig (4) Web application architecture

allows data of existing planetary design to be in different tables like Aerothermal, Geometry, TPS, etc. to be linked via a column e.g., flight vehicle. The database manager allows selective data retrieval through .NET web application user interface as shown in figure (5)

Atmospheric Entry Planetary Probe Vehicles Database

Planetary Body: MARS

Total Planetary Probe(s) Found: 7

Flight Vehicle	Lib ON Date	Entry Date	Planet	Body Shape	Shape Description	Reyn. Radius	Reyn. Area	Vehicle
Beagle 2	6/2/2003	12/29/2003	MARS	60	Spherical_Cone	0.417	0.67	68.46
DeepSpace 2	1/3/1999	12/3/1999	MARS	45	Spherical_Cone,Spherical_w/ 0.0875	0.096	3.67	
Opportunity	6/10/2003	7/7/2003	MARS	70	Spherical_Cone	0.66	5.52	836
Pathfinder_Spirit	12/4/1996	7/4/1997	MARS	70	Spherical_Cone	0.66	5.52	585.3
Spirit	6/10/2003	7/7/2003	MARS	70	Spherical_Cone	0.66	5.52	836
	8/20/1975	7/20/1976						

Fig (5) Atmospheric Entry Planetary Vehicle Web Application

6. Conclusions and Summary

A platform-independent standalone Matlab database-driven GUI application tool SPARTA has been developed. Designers have the flexibility to design new planetary probe geometry for trajectory analysis and also choose a probe from the database for analysis. A fourth order Runge-Kutta integration is employed for trajectory calculations. In addition, a comprehensive database of existing planetary probe designs has been developed and integrated with the GUI tool.

Empirical models of appropriate planetary atmosphere are modeled. SPARTA can be linked to Probe-Mesh, an Automatic grid generation framework for a wide range of planetary probe geometries. Stagnation point heating analysis has been modeled using the Fay-Riddell and Sutton-Grave Correlations

References

1. C. Davies, "Planetary Mission Entry Vehicles," Quick Reference Guide, Ver. 2.1, NASA-ARC, 2002.
2. J. Fay and F. Riddell, "Theory of Stagnation Point Heat Transfer in Dissociated Air" *Journal of Aeronautical Sciences* 25 (2), Feb 1958
3. K. Sutton, R.A Graves, "A General Stagnation Point Convective Heating Equation for Arbitrary Gas Mixture", NASA TR-376, 1971
4. Program Development Corp., 300 Hamilton Avenue, Suite 409, White Plains, NY 10601, *GridPro/az3000 User's Guide and Reference Manual*, 1996.